King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Information and Computer Science Department

ICS 253: Discrete Structures I

Summer Semester 2011-2012

Final Exam, Tuesday July 31, 2012.

Name:

ID#:

**Instructions**:

1. This exam consists of **ten** pages, including this page, containing **five** questions.
2. You have to answer all **five** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **equally weighed**.
5. This exam is out of **150** points.
6. You have exactly **150** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

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| --- | --- | --- |
| Question Number | Maximum # of Points | Earned/Deducted Points |
| **1** | **30** |  |
| **2** | **30** |  |
| **3** | **30** |  |
| **4** | **30** |  |
| **5** | **30** |  |
| **Total** | **150** |  |

Some Useful Formulas:

 ,  ,   ,  , 

1. (30 points) Choose the correct answer from the following choices. Note that the number of choices is not equal for all questions.
	1. The negation of the proposition: “Ahmad and Salim are present” is
		1. Ahmad and Salim are absent.
		2. Ahmad or Salim is absent.
		3. Ahmad is present and Salim is absent.
		4. Ahmad is absent and Salim is present
		5. all of the above.
	2. On the island of knights and knaves you encounter two people. *A* and *B*. Person *A* says, “*B* is a knave.” Person *B* says, “At least one of us is a knight.” Then,
		1. *A* is a knave, *B* is a knave.
		2. *A* is a knight, *B* is a knight.
		3. *A* is a knight, *B* is a knave.
		4. *A* is a knave, *B* is a knight.
		5. one of them cannot be determined for sure whether he is knave or knight.
	3. Suppose the variable *x* represents students and *y* represents courses, and:

*U*(*y*): *y* is an upper-level course *M*(*y*): *y* is a math course

*F*(*x*): *x* is a freshman *A*(*x*): *x* is a part-time student

*T*(*x*, *y*): student *x* is taking course *y*.

Then, the statement “Every part-time freshman is taking some upper-level course” is formulated by

* + 1. *x**y*[*U*(*y*)  *T*(*x**y*)].
		2. *y**x*[*U*(*y*)  *T*(*x**y*)].
		3. *x**y*[(*F*(*x*)  *A*(*x*))  (*U*(*y*)  *T*(*x**y*))].
		4. *y**x*[(*F*(*x*)  *A*(*x*))  (*U*(*y*)  *T*(*x**y*))].
		5. more than one answer above.
	1. *A*  (*B*  *C*) is equivalent to
		1. (*A*  *C*)  (*A*  *B*).
		2. (*A*  *B*)  (*A*  *C*).
		3. (*A*  *B*)  C.
		4. *A*  (*B*  *C*).
		5. more than one answer above.
	2. Suppose *A*  *x**y* and *B*  *x**x*. Then, it is true that
		1. *x*  *B*.
		2.   *P*(*B*).
		3. *x*  *A*  *B*.
		4.  *P*(*A*)   *P*(*B*) 
		5. more than one answer above is true.
	3. *f*  **R**  **R** where *f*(*x*)  *x*2is
		1. a function that is one to one but not onto.
		2. a function that is onto but not one to one.
		3. a function that is one to one and onto.
		4. a function that is neither one to one nor onto.
		5. not a function.
	4. $\bigcap\_{i=1}^{\infty }\left[-1-\frac{1}{i},1+\frac{1}{i}\right]$=
		1. (– 1,1).
		2. [-1,1].
		3. (– 2,2).
		4. Φ.
		5. none of the above.

In questions 8 and 9 below, suppose *g*  *A*  *B* and *f*  *B*  *C* are functions where

*A*  1234, *B*  *a**b**c*, *C*  2810,

and *g* and *f* are defined by

*g*  (1*b*)(2*a*)(3*b*)(4*a*) and *f*  (*a*8)(*b*10)(*c*2).

* 1. *f*  *g* =
		1. (110)(28)(310)(48)
		2. 8,10
		3. {1,2,3,4}.
		4. (22)(88)(1010).
		5. not well-defined.
	2. *f*  *f*1 =
		1. (110)(28)(310)(48)
		2. 8,10
		3. {(a,a).(b,b),(c,c)}.
		4. (22)(88)(1010).
		5. not well-defined.
	3. $\sum\_{i=1}^{n}\sum\_{j=1}^{i}\left(ij\right)$=
		1. $\sum\_{i=1}^{n}i^{2}$.
		2. $\sum\_{i=1}^{n}i^{3}$.
		3. $\sum\_{i=1}^{n}\frac{i^{3}+i^{2}}{2}$.
		4. $\sum\_{i=1}^{n}i^{4}$.
		5. none of the above.
1. (30 points) Induction and Recursion
	1. (15 points) Prove that *n*2 – 1 is divisible by 8 whenever *n* is an odd positive integer.
	2. (15 points) Prove that the number of leaf nodes in a full binary tree is one more than the number of internal nodes.

(**Note**: A full binary tree is a tree in which each node has either two children or zero children.)

1. (30 points) Counting
	1. (5 points) How many positive integers less than 1000 are divisible by 7 but not by 11? Show the details of your answer.
	2. (5 points) There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.
	3. (10 points) The English alphabet contains 21 consonants and 5 vowels. How many strings of eight lowercase letters of the English alphabet contain at least two vowels? Show the details of your answer.
	4. (10 points) Give a formula for the coefficient of *xk* in the expansion of $\left(x+\frac{1}{x}\right)^{100}$. Show the details of your answer.
2. (30 points) Discrete Probability
	1. (5 points) Find the probability of selecting none of the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 64? Show the details of your answer.
	2. (10 points) What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails? Show the details of your answer.
	3. (15 points) Show the details of your answers below.
		1. (3 points) What is the probability that two people chosen at random were born during the same month of the year?
		2. (5 points) What is the probability that in a group of *n* people chosen at random, there are at least two born in the same month of the year?
		3. (7 points) How many people chosen at random are needed to make the probability greater than 1/5 that there are at least two people born in the same month of the year?
3. (30 points) Recurrence Relations and Their Solution
	1. (8 points) Find a recurrence relation for the number of bit strings of length *n* that contain three consecutive 0’s.
	2. (7 points) Prove that the sequence $a\_{n}=2 .4^{n}+3n4^{n}$ is a solution of the recurrence relation $a\_{n}=8a\_{n-1}-16a\_{n-2}$.
	3. (15 points) Find the solution to the recurrence $a\_{n}=5a\_{n-2}-4a\_{n-4}$ with $a\_{0}=3, a\_{1}=2, a\_{2}=6 and a\_{3}=8 $.